Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 9: Linear Inequalities

Definition 1. Inequality *When we replace the equal sign (=) in an equation with any of the four inequality symbols $<$, $\leq$, $>$, $\geq$, then the resulting expression is an inequality.*

Definition 2. Domain *The domain of a variable in an inequality is the set of all real numbers for which both sides of the inequality are defined. For example, the domain of $\sqrt{x+1} < x$ is all real numbers greater or equal to -1. In interval notation, the domain is $[-1, \infty)$.*

Definition 3. Solving Linear Inequality *Solving linear inequality in one variable is similar to solving a linear equality in one variable.*

Example 1. *Solve the inequality:* $7x - 11 < 2(x - 3)$
Solution:

$$\begin{aligned} 7x - 11 &< 2(x-3) \\ 7x - 11 &< 2x - 6 \\ 7x - 11 + 11 &< 2x - 6 + 11 \\ 7x &< 2x + 5 \\ 7x - 2x &< 2x + 5 - 2x \\ 5x &< 5 \\ x &< 5 \end{aligned}$$

Then the solution set is $\{x : x < 5\}$, or in interval notation $(-\infty, 5)$.

Example 2. *Solve:* $8 - 3x \leq 2$
Solution:

$$\begin{aligned} 8 - 3x &\leq 2 \\ 8 - 3x - 8 &\leq 2 - 8 \\ -3x &\leq -6 \\ \frac{-3x}{-3} &\geq \frac{-6}{-3} \qquad \text{(Dividing by } -3\text{, Reverse the direction of the inequality symbol)} \\ x &\geq 2 \end{aligned}$$

So the solution set is $\{x : x \geq 2\}$, or in interval notation $[2, \infty)$.

Definition 4. Compound Inequality *The combination of two or more inequalities is called a compound inequality.*

Example 3. *Write the solution set of the compound inequality:*

$$2x + 7 \leq 1 \quad \underline{or} \quad 3x - 2 < 4(x - 1)$$

Solution:

$$2x + 7 \leq 1 \qquad \underline{or} \qquad 3x - 2 < 4(x - 1)$$

$$
\begin{aligned}
2x+7-7 \le 1-7 \quad & \underline{or} \quad 3x-2<4x-4 \\
2x \le -6 \quad & \underline{or} \quad 3x-2+2-4x<4x-4+2-4x \\
2x \le -6 \quad & \underline{or} \quad -x<-2 \\
x \le -3 \quad & \underline{or} \quad x>2 \quad \underline{\textit{Multiplying by } -1 \textit{ reverse the direction of the inequality symbol}}
\end{aligned}
$$

The solution set of the compound inequality is $(-\infty,-3]\cup(2,\infty)$.

Example 4. *Write the solution set of the compound inequality:*

$$2(x-3)+5<9 \quad \underline{and} \quad 3(1-x)-2 \le 7$$

Solution:

$$
\begin{aligned}
2(x-3)+5<9 \quad & \underline{and} \quad 3(1-x)-2 \le 7 \\
2x-6+5<9 \quad & \underline{and} \quad 3-3x-2 \le 7 \\
2x-1<9 \quad & \underline{and} \quad 1-3x \le 7 \\
2x<10 \quad & \underline{and} \quad -3x \le 6 \\
x<5 \quad & \underline{and} \quad x \ge -2 \quad \underline{\textit{Dividing by } -3 \textit{ reverse the direction of the inequality symbol}}
\end{aligned}
$$

The solution set of the compound inequality is $(-\infty,5)\cap[-2,\infty)=[-2,5)$.

Remark 1. *Note that* $(or=\cup)$, *and* $(and=\cap)$.